Mathematics, measurement and experience of rotations around 3 axes

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Abstract. An amusement park is full of examples, that can be made into challenging problems for students, combining mathematical modelling with video analysis, as well as measurements in the rides. The authenticity of the tasks in enjoyable situations often leads to inspiring and enlightening discussions. This paper focuses on an amusement ride involving rotations around three axes, and could offer a large-scale illustration for the study of dynamics in three dimensions. As a first approximation, the different motions can be treated separately, but the combined rotations is found to lead to a relatively large Coriolis effect. The forces on the rider can be measured with comoving accelerometers. The data, as well as video recordings of the motion, reveal that the main rotation is not exactly uniform, giving an additional contribution, needed to explain the size of the maximum forces.

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1. Introduction

Traditional amusement-ride related textbook problems include free fall, circular motion, pendula and energy conservation in roller coasters, where the moving bodies are typically considered point-like. However, an amusement park can offer many more examples that are useful in physics and engineering education, many of them with a strong mathematical or content [1-7]. These examples are well-suited for modelling and the results can be compared to authentic accelerometer data, as well as to data from photos and video clips. Today's smartphones include 3D accelerometers and gyros, giving students easy ways to measure acceleration and rotation, using e.g. the app Physics Toolbox Roller Coaster [8, 9] or PhyPhox [10, 11]. The interpretation of the data offers good practice in identifying coordinate axes [1, 6, 7].

Motion in three dimensions is common in engineering applications. Describing three-dimensional motion including combinations of acceleration and rotation is a challenge where mathematics plays a central role. For several years, I had the privilege of running problem-solving classes for the mechanics courses in the spring for first-year engineering physics students, who used the textbooks by Meriam and Kraige [12]. For dynamics problems with rotations in three dimensions around several axes, intuition based on everyday experiences is of little help. I observed as the course progressed how students, one by one, learned to trust the mathematics and displayed happy confidence when they were able to solve these challenging problems. As students struggle to visualize generalized 3D motion, including rotating arms, links, shafts, cranks and disks, real-life concrete examples can be useful, such as rotating pendulum rides [1, 7]. The Star Shape ride Mechanica (figure 1), which opened at Liseberg in 2015 would have been another excellent example.

Figure 1. The Star Shape ride Mechanica at Liseberg. The 12 m main arm rotates a full turn in 8 s. At the same time the star, with a radius of 4 m, rotates a full turn in 10 s. Each of the gondolas in the star seats 5 guests, and can rotate freely around its axis, as discussed in more detail in section 3.

2. Forces in circular motion

Students are familiar with centripetal acceleration, and usually understand reasonably well the forces acting in an ordinary carousel moving in uniform circular motion in a horizontal plane. In the common chain flyer rides, the acceleration required for the circular motion is directly visualised by observing the angles of the chains to the vertical [13].

If the motion is, instead, in the vertical plane, as in Ferris wheels and Flying carpets, most students have learned in high-school how to work out the forces at the top and bottom, but lack systematic ways to describe what happens half-way up or down, or in more generally chosen points. These rides offer opportunities to connect the mathematical description with the experiences of their bodies in the ride. Textbooks often fail to make this connection; forces usually act on inanimate objects.

Circular motions in vertical planes come in many different versions. In ferris wheels and flying carpets, your own body does not rotate, but the experience still differs depending on your own orientation relative to the plane of motion. If your own body also takes part in a rotation that brings you upside down, it becomes even more obvious that our bodies are not point particles. This rotation and the associated forces also show up in electronic data collected during a ride, inviting students to relate the data to the experiences of their own body.

Motion in a system rotating relative to an inertial system leads to the Coriolis effect. This can be observed even in a small children's carousel - or even better in a slowly rotating observation tower or loading platform - by bringing a small, soft object on a string, as a miniature "Foucault pendulum" [14, 15]. The Coriolis effect is found to give a measurable contribution to the forces in the Star Shape ride.

3. The Star Shape Ride Mechanica - combining rotations around three axes

Figure 1 shows the Star Shape ride from Zierer [16] which could be an excellent example for students studying rotations in three dimensions. The ride first makes three full turns, each taking about $T = 8\,\mathrm{s}$, moving the center of the star in a large circle with radius $R = 12\mathrm{m}$ in the vertical plane, while the star rotates slowly ($T_{star} \approx 10\,\mathrm{s}$ per turn) around the main arm. The motion stops for a while in the highest point, where the star changes its directions of motion and then the main arm makes a few full turns in the other direction. The gondolas of each arm can swing freely throughout the ride.

Figure 2 shows the accelerometer data during a ride in one of the seats furthest away from the centre of the star. The data were collected using a Wireless Dynamic Sensor System (WDSS) [17] but could also have been obtained using a smartphone [8, 10]. The force from the ride on the rider needs to compensate for gravity, while also providing the force required for the acceleration. The biomechanical effect of the forces is defined in the rider-based coordinate system where "vertical" denotes the direction along the spine towards the head, "longitudinal" is in the forward direction and the "lateral" direction is from right to left as discussed in more detail in earlier work [18].

To understand the the accelerometer graphs, we first consider the different motions separately.

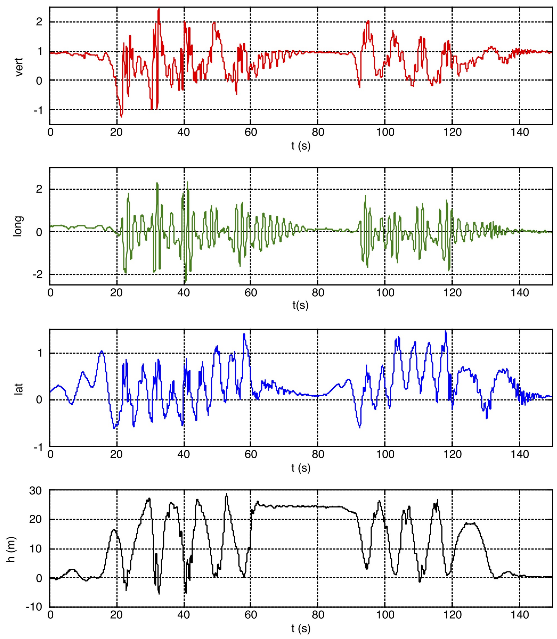

Figure 2. Accelerometer data for a rider in one of the outer seats of the Star Shape ride Mechanica collected with the WDSS sensor system [17]. The sensor was carried in a vest on the body. The data shows the components of the vector $(\mathbf{a}-\mathbf{g})/|g|$ for the three axes in the coordinate system of the rider, where "vertical" denotes the direction along the spine towards the head, "longitudinal" is in the forward direction and the "lateral" direction is from right to left The final graph shows the elevation obtained from the WDSS (which converts the change in air pressure to elevation).

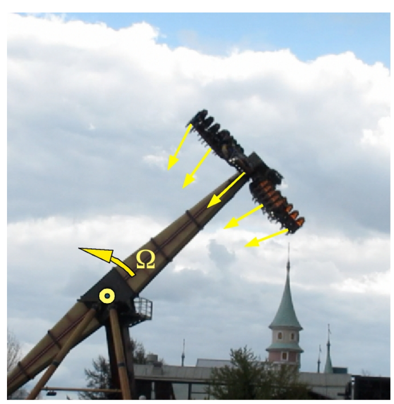

Figure 3. The Star shape ride viewed from the side. The arrows mark the centripetal acceleration due to the main rotation, $\mathbf{\Omega}$, for a few different rider positions.

3.1. The main rotation

Figure 3 shows the ride viewed from the side. The main rotation with an angular velocity $\Omega = 2\pi/T \approx 0.79$ rad/s leads to a centripetal acceleration for the rider. As shown in figure 3, it is mainly along the direction of the main arm, but may also have a component in the plane of the star depending on the position of the rider. If the angular velocity of the main rotation is constant, the acceleration of the centre of the star is given by $a_c = R(2\pi/T)^2 \approx 0.75g$. The accompanying centripetal acceleration towards the centre of the star can reach $0.25g$ for a rider located 4m from the centre.

3.2. Freely rotating gondola

The free pitch rotation of the gondola, leads to a mixing of the vertical and longitudinal coordinates. This is best observed in the data when the main rotation stops with the star in the highest position (about $60s - 75\,\mathrm{s}$ in the graph), showing damped oscillations around the equilibrium orientation. If θ denotes the angle between the vertical axis of the rider and the acceleration of gravity, $\mathbf{g}$, the force in the forward direction can be written as $mg\sin\theta$, changing sign during the swinging to and fro, whereas the force in the *vertical* direction varies between unity and $mg\cos\theta$. Normally, one would expect larger values as a pendulum passes the lowest point, but in this case, the sensor is mounted close to the rotation axis, where the centripetal acceleration is very small. In the discussions below, we neglect the rotation of the gondola, but keep in mind that the vertical and longitudinal values will be mixed in the data. Figure 4 shows the length of the combined vertical and longitudinal components, that may also be used for comparison with calculated values.

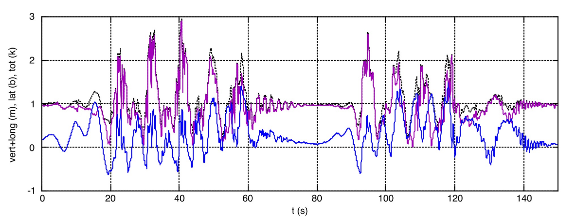

Figure 4. Since the seats with the riders can rotate around the arm of the star (the lateral axis), the longitudinal and vertical components are mixed depending on the orientation of the seat. The lower graph (blue) shows the lateral component of the G force $(\mathbf{a} - \mathbf{g})/|g|$, whereas the upper continuous graph (magenta) shows the length of the combined vertical and longitudinal components. The dotted line shows the size of the total G force.

3.3. The rotation of the star

The rotation of the star, with a period $T_{star} \approx 10\,\mathrm{s}$ leads to an acceleration in the y direction: $a_{y,star} = d\omega^2 \leq 0.16g$, where $\omega = 2\pi/T_{star} \approx 0.63\mathrm{rad/s}$ and $d \leq 4m$ is the distance between the rider and the centre of the star. During the time $60 - 85\,\mathrm{s}$ in the graph the centre of the ride is stationary, with the star in the highest point, while the rotation slows down and then the star begins to rotate in the other direction. We would thus expect the lateral component to drop from $0.16g$ to zero and then increase again as seen in the graph. (The data for the lateral component shown in figure 2 takes slightly

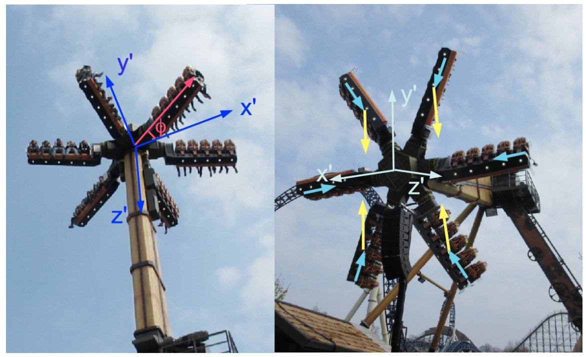

Figure 5. Centripetal accelerations in the plane of the star. The rotation of the star leads to an acceleration towards the centre of the star (shown in light blue arrows), whereas the main rotation (around the x' axis) only causes centripetal acceleration components orthogonal to that axis as shown in figure 3. The component in the plane of the star due to the main rotation is shown by (yellow) arrows, pointing up or down (i.e along the y' axis in the figure) and the length is proportional to the distance to the x' axis

larger values, both as it slows down, after 60 s and as it starts to move in the other direction after 80 s. In addition, the data does not come down all the way to zero. This indicates that the orientation of the sensor is is not perfect.)

Depending on your position and the orientation of the star, the main rotation can also lead to an acceleration in the y direction, i.e. toward the centre of the star, up to $a_{y,main} = d\Omega^2 \leq 0.25g$. The centripetal acceleration in the y direction can thus reach $0.41g$. Figure 5 shows the components in the plane of the star of these two centripetal accelerations.The accelerometer data also account for the force required to counteract gravity, and we would expect to see a readings between $-0.6g$ and $1.4g$ for the lateral component, in good agreement with the data in figure 2.

3.4. The "vertical" acceleration and the Coriolis effect

The main rotation is expected to give a centripetal acceleration $a_{c,main} \approx 0.75g$, as discussed above. This acceleration is directed along the main arm, which will be a combination of the longitudinal (x) and vertical (z) axis of the rider. From this motion, we could expect to see a maximum force of $1.75mg$ in the positive or negative x or z directions. However, the graphs show larger values, and it is necessary to look more closely into the vertical and longitudinal forces.

The rotation of the star causes a rider to have a velocity $\mathbf{v}'$ relative to the centre for the star. The main rotation $\mathbf{\Omega}$ leads to a rotation of the coordinate system of the star, resulting in an additional acceleration

$$\mathbf{a}_{Cor} = 2\mathbf{\Omega} \times \mathbf{v}'$$

which is orthogonal both to the velocity relative to the centre of the star, and to the main rotation axis and can reach $a_{Cor} = 2d\Omega\omega \approx 0.40g$ depending on the position of the rider. When the star is at the bottom, this acceleration is in the vertical direction, $0.40g$ upwards on one side and downwards on the other. This combines with the force required for the main rotation and to compensate for gravity, giving a total of $2.15g$, still slightly smaller than the largest of the measured values shown in Figure 2.

Figure 6. The motion of a rider passing the lowest point of the ride, while rotating around the arm of the star. The arrows mark the "up" direction for the rider and the time interval between the screen shots is 0.4s.

3.5. Additional corrections

The pendulum motion of the seats around the arm of the star (figure 6) might have been expected to contribute to the vertical acceleration. However, as seen from the data for pendulum motion when the ride is in the highest point, this effect is very small ? the sensor is located close to the rotation axis of the gondola.

To reconcile measured data with theoretical expectations, a video was recorded from the side, to look into more details of the main rotation. It was found that the upper half of the turn is slightly longer than the lower half, leading to a larger centripetal acceleration when the riders are in the lowest position, and thus slightly larger values for the interaction. This can also be observed from the elevation graph in figure 2.

3.6. Comparison between theory and experiment

The example of the Star Shape ride shows how the data from the ride can be compared with theoretical expectations at different degrees of complexity. During introductory courses, looking into one motion at a time, will be sufficiently challenging. Student groups seeking additional difficulties may choose to push the accuracy in the comparison. When students have learned to work with rotations in three dimensions, and practiced transformation between different coordinate systems, it could be an interesting exercise to write a small program to calculate the position as well as the forces on the rider, for comparisons with measured data, along the lines indicated in the Appendix.

3.7. Angular velocities around three axes

Many smartphones can measure angular velocities around the three axes of the phone [1,6-11], referred to as pitch, yaw and roll. The pendulum motion of the gondola

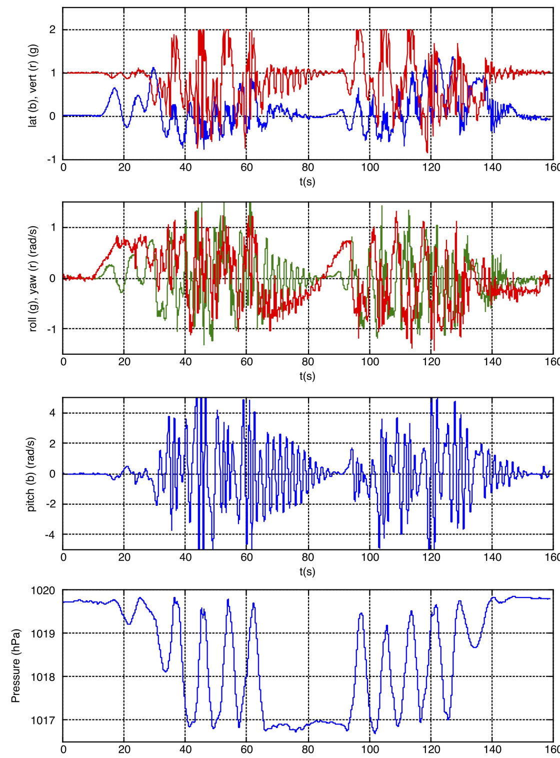

Figure 7. Accelerometer, rotation and air pressure data obtained from a Nexus 5 using Physics Toolbox Roller Coaster [8]. The "g force" in the longitudinal direction, similar to that shown in figure 2, was omitted from the plot. The accelerometer sensor of the phone is limited to $\pm 2g$. (This range would have been sufficient for a smaller Coriolis effect and main rotation with constant angular velocity.)

corresponds to pitch, whereas turning right or left is defined as yaw and leaning left or right is a roll. Figure 7 shows accelerometer, rotation and air pressure data obtained from a Nexus 5. (The phone was kept in a closed pocket, and rested on the thigh, aiming to keep the phone as well aligned as possible. No reorientation of the axes was performed after the data collection.)

The star rotates with an angular velocity up to $\omega_{star} = 2\pi/T_{star} \approx 0.63$rad/s. At the beginning of the ride, the riders move forward, giving a positive yaw value. As the ride stops in the highest point, the gondolas have reversed their direction relative to the main arm, and the riders instead move backwards until the star reverses its direction of motion.

As the main arm of the ride begins to move, the star begins to tilt. A positive roll then makes the rider lean to the right, and the ride needs to exert a force from the right on the rider, corresponding to a positive "lateral" force as seen e.g. in the beginning of the graphs.

The vertical accelerometer data in the highest point fall below $1g$ as the gondola swings back and forth in the damped oscillation. The maximum pitch value related to this motion can be estimated as $\theta_0(2\pi/T_p)$ where T_p is the period of the pendulum, close to 2s, and a value of the maximum angle θ_0 can be obtained from the vertical accelerometer graph. During the first oscillation the vertical acceleration drops to close zero, corresponding to an angle close to 90°, whereas later oscillations are smaller. (The pitch values are slightly smaller than could be expected, but we can also note that the roll values at the highest point should have been zero for a perfectly aligned device.)

Angular velocities around three axes is a challenging encounter in dynamics. Amusement rides showing the different types of rotations involving your own body can be one way to get students more familiar with the concepts pitch, yaw and roll and the mathematics of rotating coordinate systems, as they, themselves - or a friend - is rotating.

4. Discussion

Amusement rides offers many possibilities for mathematical modelling with rich opportunities for interesting projects, where students can work on problem formulations, discussions and revisions of their understanding, as well as evaluations of the results. The StarShape ride involving rotations around three axes demonstrates that amusement park examples extend far beyond introductory observations of free fall, energy conversions and centripetal forces. Today's easily accessible video footage and sensor data from smartphones can challenge the mathematical models developed, and support successive refinement of the models.

Acknowledgments

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Appendix: Mathematical description of the Star Shape motion

The Star Shape ride is suitable for small programming exercises. A first step is to describe the position of a rider sitting in one of the gondolas in the ride using a location **r**' a velocity **v**' and an acceleration **a**' in the *non-inertial reference frame* of the star, which is rotating with angular velocity Ω due to the main rotation of the ride. These can be related to a location **r**, a velocity **v** and an acceleration **a** relative to the inertial frame of reference, as illustrated in figure 9 and described in more detail below.

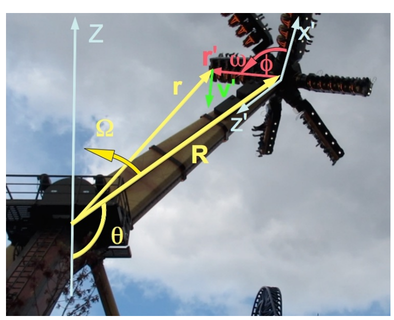

Figure 8. Coordinate axes and angles that may be used to describe the motion of the Star Shape ride.

Transformations between the different coordinate systems of interest

A first step is to consider the main rotation. The location **R** of the centre of the star can be defined in the stationary coordinate system where the X-axis defines the rotation, and is defined to point to the south and the Z axis is vertical. We define the angle θ to

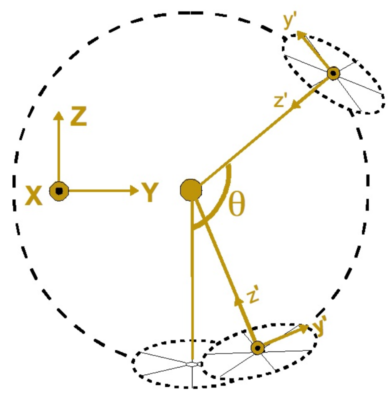

Figure 9. Coordinate systems used for the Star Shape ride.

be zero when the star is in the lowest position (figure)

$$R_X = 0$$
$$R_Y = R\sin\theta$$
$$R_Z = -R\cos\theta$$

The location of the center of the star can be written as $\mathbf{R} = R_X\mathbf{i} + R_Y\mathbf{j} + R_Z\mathbf{k} = (R_X, R_Y, R_Z) = R(0, \sin\theta, -\cos\theta)$, where **i**, **j** and **k** are the unit vectors in a fixed coordinate system.

We next describe the motion of a person in one of the gondolas in the moving $x'y'$ plane defined by the star. The axes are chosen so that the z' axis points towards the center of the ride and the x' and X axes coincide.

$$x' = d\cos\phi$$
$$y' = d\sin\phi$$
$$z' = 0$$

or $\mathbf{r}' = d(\cos\phi e'_x + \sin\phi e'_y) = de'_r$.

The velocity within the star coordinate system can be written as given by $\mathbf{v}' = v'(-\sin\phi e'_x + \cos\phi e'_y) = v'e'_\phi$, where the speed is given by $v' = d\dot{\phi}$. (Note that the direction of the x' axis (e'_x) does not change during the motion.)

We will also use coordinates in the system of the rider. "Lateral" (sideways) G forces are defined to be positive for a force from the right, corresponding to $-e'_r$. As long as the person in the gondola looks onto the next gondola, the "longitudinal" direction coincides with e'_ϕ and the "vertical" direction coincides with e'_z. Since the orientation of the gondola shows a somewhat chaotic behaviour, we can also consider the magnitude of the vector sum of the vertical and longitudinal components, although the rotation of the gondolas will also introduce additional "vertical" forces.

Rotating coordinate axes

The main rotation leads to a rotation of the coordinate system of the star

$$\begin{aligned} e'_x &= \mathbf{i} \\ e'_y &= \cos\theta\,\mathbf{j} + \sin\theta\,\mathbf{k} \\ e'_z &= -\sin\theta\,\mathbf{j} + \cos\theta\,\mathbf{k} \end{aligned}$$

Using these axes we can also rewrite the co-moving coordinates, e_r and e'_ϕ of the rider in terms of stationary coordinates

$$\begin{aligned} e'_r &= \quad \cos\phi\,\mathbf{i} + \sin\phi\cos\theta\,\mathbf{j} + \sin\phi\sin\theta\,\mathbf{k} \\ e'_\phi &= -\sin\phi\,\mathbf{i} + \cos\phi\cos\theta\,\mathbf{j} + \cos\phi\sin\theta\,\mathbf{k} \\ e'_z &= -\sin\theta\,\mathbf{j} + \cos\theta\,\mathbf{k} \end{aligned}$$

Using these coordinate axis, the location of a rider in the star , can be expressed in the fixed coordinate system as $\mathbf{r}' + \mathbf{R}$, giving the expressions

$$\begin{aligned} x_s &= d\cos\phi \\ y_s &= d\sin\phi\cos\theta + R\sin\theta \\ z_s &= d\sin\phi\sin\theta - R\cos\theta \end{aligned}$$

Angular velocities

Angular velocities can be introduced to express the time dependence of the angles, $\theta = \Omega t$ and $\phi = \omega t + \phi_0$, where ϕ_0 depends on the choice of gondola. Expressions for the velocities and accelerations in the stationary coordinate system can be worked out by taking the time derivatives of the positions coordinates.

The velocity components are given by

$$\begin{aligned} v_x &= -d\,\omega\sin\phi \\ v_y &= d(\omega\cos\phi\cos\theta - \Omega\sin\phi\sin\theta) + R\Omega\cos\theta \\ v_z &= d(\omega\cos\phi\sin\theta + \Omega\sin\phi\cos\theta) + R\Omega\sin\theta \end{aligned}$$

Here, the terms proportional to ω correspond to the velocity of the rider relative to the coordinate system defined in (1-1) above, $\mathbf{v}' = d\,\omega e'_\phi = d\,\omega(-\sin\phi e'_x + \cos\phi e'_y)$. The additional terms for v_y and v_z, proportional to Ω, arise through the rotation of the coordinate system of the star. The last terms, describe the motion of the center of the star.

The acceleration (with respect to a fixed coordinate system) is obtained by taking the time derivatives of the velocity expression above. In addition to the familiar expressions of the individual centripetal accelerations, $-\mathbf{R}\Omega^2$ and $-\mathbf{r}'\omega^2$, the rotation of the coordinate system of the star leads to an additional acceleration term:

$$\mathbf{a}_{cor} = 2\boldsymbol{\Omega} \times \mathbf{v}'$$

Since $\mathbf{\Omega} = \Omega e_X = \Omega e'_x$ and $\mathbf{v}' = v' e_\phi = d\,\omega(-\sin\phi e'_x + \cos\phi e'_y)$ this vector product will be in the z' direction:

$$\mathbf{a}_{cor} = \Omega v' \cos\phi\, e'_z = 2d\,\Omega\omega\cos\phi\, e'_z \tag{1}$$

thus contributing up to $0.40g$ in the positive or negative z' direction.

Forces on the rider

The total force on the rider can be expressed as $m\mathbf{g} + \mathbf{X} = m\mathbf{a}$ where $\mathbf{X} = m\mathbf{a} - \mathbf{g}$ is the force from the ride from the ride on the rider, related to the components of the acceleration as

$$X_x/m = \ddot{x}_s$$
$$X_y/m = \ddot{y}_s$$
$$X_z/m = g + \ddot{z}_s$$

where $g = |\mathbf{g}| \approx 9.82\mathrm{m}/s^2$. The "G force" can be defined as $\mathbf{G} = \mathbf{X}/mg = (\mathbf{a} - \mathbf{g})/g$, and is independent of the mass of the rider.

Forces in the coordinate system of the rider

For the rider, it is more interesting to know the forces in the coordinate system moving together with the rider which would also be the forces measured by a comoving accelerometer, e.g. in a smartphone [8, 10].

The lateral component is defined by the direction e'_r in (1), $X_{lat} = -\mathbf{X} \cdot e_r{}'$. If we neglect the individual rotation of the gondola arms, the coordinate in the direction of the motion within the star, $\mathbf{v}'$, corresponds to the "longitudinal" component, i.e. $X_{long} = \mathbf{X} \cdot e'_\phi$. Finally, the "vertical" component is along the z' axis, i.e. $X_{vert} = \mathbf{X} \cdot e'_z$.

Figure 10 shows the result of the theoretical model (sitting at the maximum distance, 4m, from the center), in a gondola starting with a phase $\phi = -90^o$ at $t = 0$), which can be compared to the experimental data in figure 2. Since the lateral and vertical components are mixed when the gondola rotates around the arm of the star, figure 10 shows the results with the lateral and vertical components combined, as well as the total force, as obtained from the equations in this appendix.

It can be noted that the maxima and minima for the different forces agree with the simpler treatment in section 3, whereas the maximum total force from the measurements is slightly larger. Comparing the results for the elevation shows that the main rotation for the Star Shape is not uniform - as found also from a video of the motion.

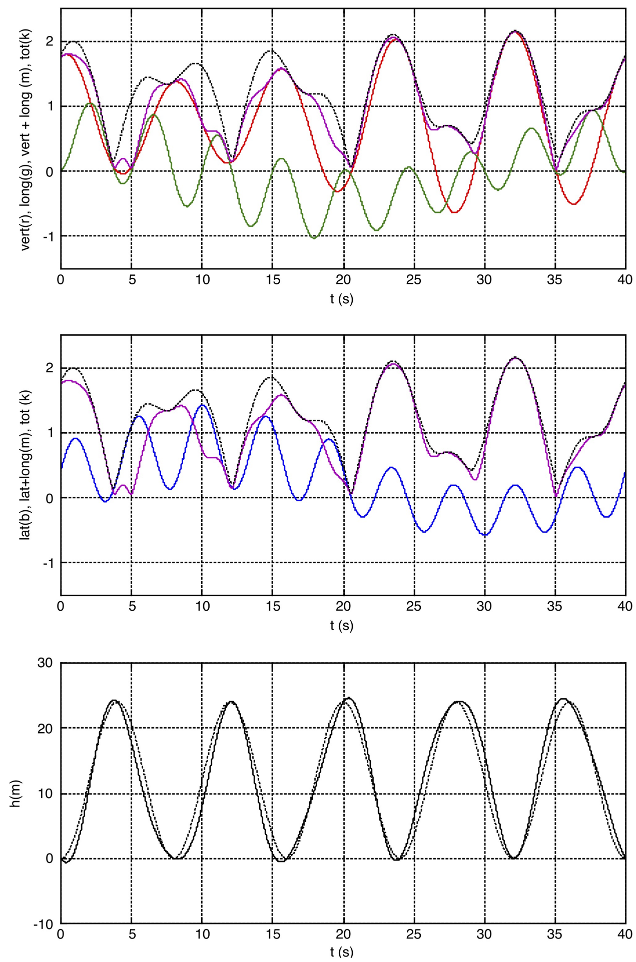

Figure 10. Calculated values for the different components of the forces. To account for the rotation of the seats around the lateral axis, the calculated vertical and longitudinal components have been combined, enabling a more direct comparison with the graph from the experimental data in figure 4. The first graph shows also the vertical and longitudinal components separately, and the middle graph shows the lateral component together with the combination of the two other components, as well as the total size of the force from the ride acting on the rider. The final graphs shows the elevation of the center of the star (solid) and of a rider at maximum distance rom the center of the star (dotted).