Acceleration and rotation in a pendulum ride, measured using an iPhone 4.

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Abstract. Many modern cell phones have built-in sensors that may be used as a resource for physics education. Amusement rides offer examples of many different types of motion, where the acceleration leads to forces experienced throughout the body. A comoving 3D-accelerometer gives an electronic measurement of the varying forces acting on the rider, but a complete description of a motion, also requires measurement of the rotation around the three axes, as provided e.g. by the iPhone 4. Here we present and interpret accelerometer and gyroscope data that were collected on a rotary pendulum ride.

Figure 1. The Rockin' Tug family ride combines a pendulum and circular motion

1. Introduction

Inertial motion tracking in three dimensions requires accelerometer and gyroscopic data around three axes. The ability to track motion has many applications [1], including movie making, crash analysis, sports, virtual reality - and games. The iPhone 4 offers a user-friendly interface to a motion tracking MEMS (MicroElectroMechanical System) sensor, offering new possibilities not only for games but also for the physics classroom, where the data collection should be combined with an analysis of the motion studied.

Amusement parks are among the most favourite school trips. In an amusement park the visitor may experience many examples of acceleration and rotation from simple children carousels rotating around a vertical axis, to looping and twisting roller-coasters, with changing acceleration and rotation in three dimensions. The rides can be studied at many different levels, from observation of motion and the interplay between kinetic and potential energies in pendulum rides and roller coasters, to data collection and detailed analysis of the rides.

The pendulum is a classic textbook example, which can be studied using ordinary playground swings [2] and also in many amusement park rides. In this paper we focus on a pendulum ride, with an added rotation: the "Rockin' Tug" family ride from Zamperla et al [3]. The relatively simple motion enables a mathematical description of the motion. At the same time, the combination of pendulum and rotation around an additional axis leads to non-zero results for all axes, both for the accelerometer and gyroscopic sensors. As seen from figure 1 the pendulum string is replaced by a rail, which is known to have a radius of R=11.5m. The boat moves along the rail and can also rotate around its own axis with a rotation speed 11 turns per minute according to specifications. How do the combined pendulum and rotation motions combine? What are the resulting forces on the rider - and how can we understand the data from the iPhone?

2. The pendulum motion

The accelerometer data from the Iphone, which was held in place on the seat of the ride, are shown in figure 2. Since the the sensors move along with the rider, the directions of the axes change all the time, with the z axis pointing up from the seat. Figure 3 shows the ride and the radius of the circular arc of the rail, which takes the place of the string in a pendulum. If maximum angle of the pendulum is θ_0, the time dependence of the angle in the pendulum motion can be written as $\theta(t) = \theta_0 \cos pt$ with $p^2 = R/g$ (using the approximation valid for small angles). The vertical accelerometer data in the top graph of figure 2 shows a periodic variation, corresponding to half a period of the pendulum motion. It is consistent with the period $T = 2\pi\sqrt{R/g} = 6.8$s for a mathematical pendulum of length R= 11.5m.

It should be noted that an accelerometer does not measure components of acceleration, but of the vector **a-g**, corresponding to the force per kilogram from the ride on the rider. The downwards acceleration along the rail, caused by gravity, gives no
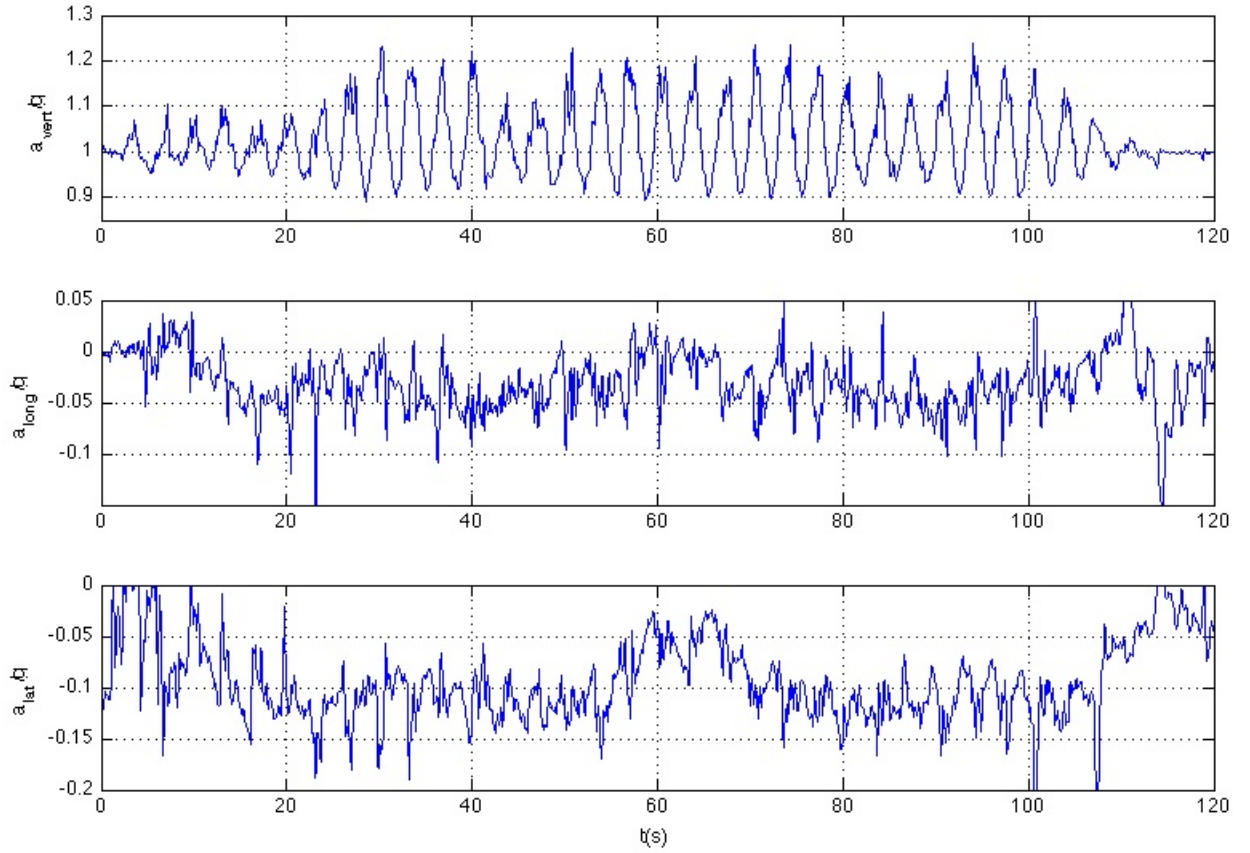
Figure 2. Accelerometer data from the Iphone, which was held in place on the seat of the ride. The "vertical" z-axis points up from the seat. The sensor was placed with the "longitudinal" x-axis pointing to the back of the rider and the "lateral" y axis pointing to the right.

contribution to the accelerometer data. For a pure pendulum ride only the vertical axis shows non-zero results. The vertical component of the accelerometer will be $g \cos \theta_0$ at the turning points and $g \left(3 - 2 \cos \theta_0 \right)$ as the pendulum passes the lowest point. From the vertical accelerometer data in figure 2, we can conclude that amplitude of the pendulum motion is about 25°. (The interpretation of accelerometer data from a swing was discussed in more detail in [2]).
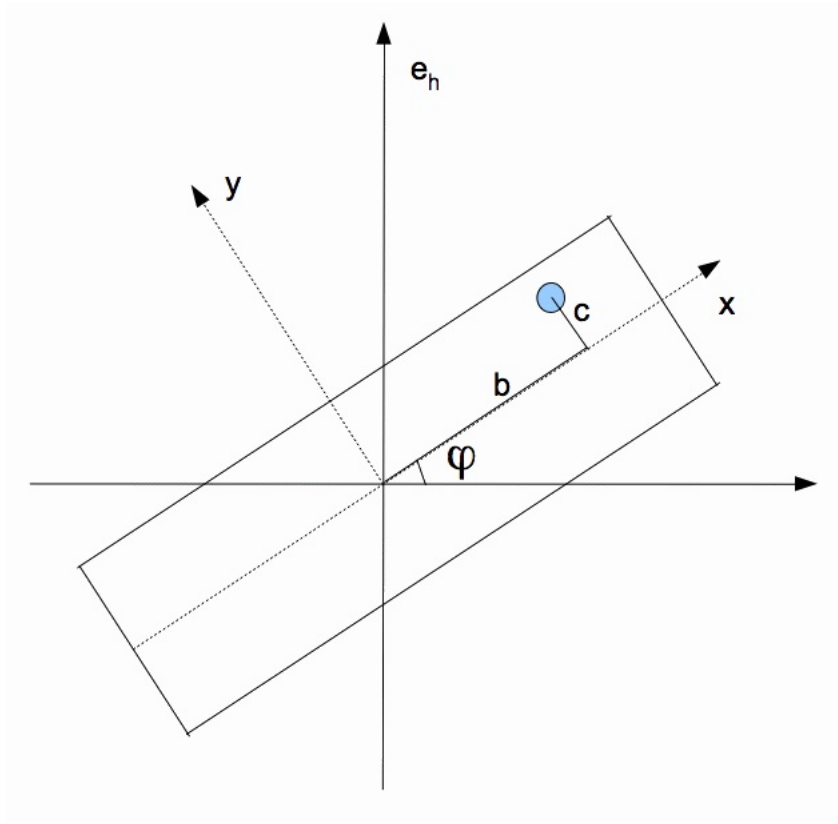
3. The rotation of the boat

After a few oscillations, the little tug starts to rotate as shown in figure 1. The rotation of the boat leads to a centripetal acceleration of the rider towards the centre of the boat, depending on the distance to the centre of rotation and on the angular velocity, Ω. Figure 4 shows the coordinates for a rider in the moving x-y coordinate system
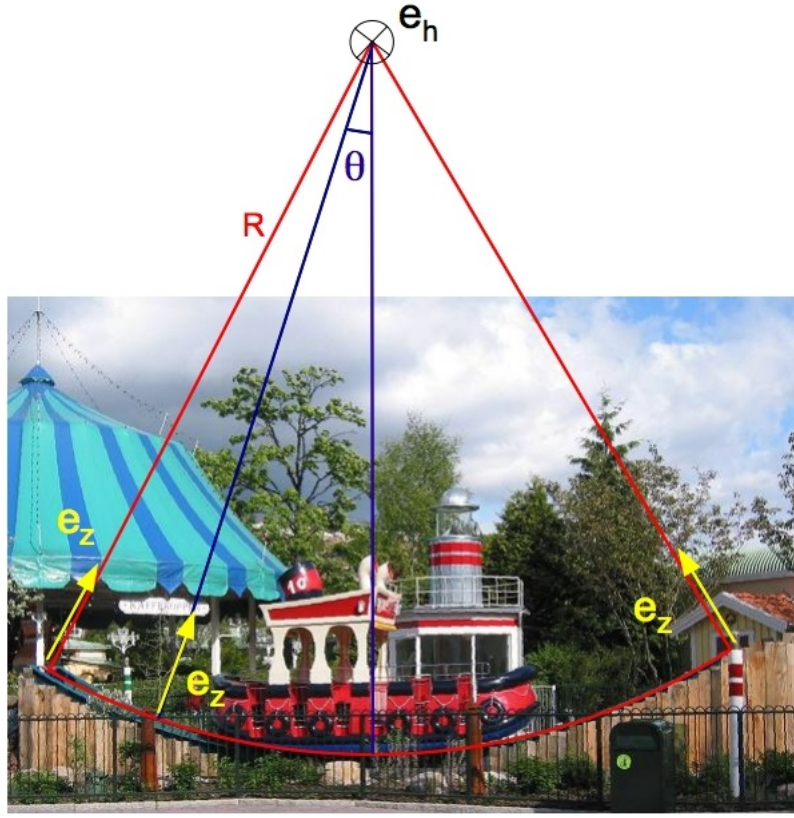
Figure 3. The radius of the circular ar of the rails (red). The blue lines mark the angle to the vertical. The horizontal axis of rotation for the pendulum motion is shown at the centre of the circle.

attached to the boat. The centripetal accelerations in x and y directions, corresponding to the "longitudinal" and "lateral" accelerations in figure 2, will thus be $a_x = -b\Omega^2$ and $a_y = -c\Omega^2$ (using the coordinates from figure 6. The specified rotation of 11 turns per minute corresponds to an angular velocity of Ω =1.15 rad/s. The accelerometer data (averaged over the times 20-50 s and 70-100 s) from 2 indicate that the sensor was placed at a point located b=0.8m in the x direction and c=0.3m in the y-direction away from the centre.

The circular motion should then be added to the pendulum motion. Figure 5, illustrates the motion of the rider during one pendulum period starting at the highest point. The shape of the combined pendulum and circular motion depends on the relation between the periods for the pendulum and the circular motion. Figure 6 shows the combined motion for four pendulum periods which corresponds to five circular motions. (The little gap left in the upper right corner marks the start and finish of the drawing.) Closed orbits result when the ratio is a rational number as in this case.

Figure 4. Coordinate system used to describe the motion of the rider relative to the centre of the boat, as well as the axis of rotation for the pendulum motion.
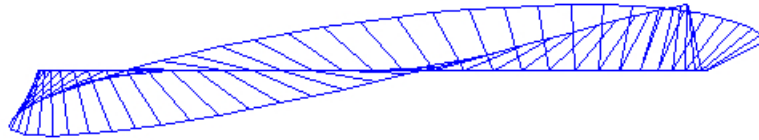

Figure 5. The motion in Rockin Tug consists of a pendulum motion, along the straight horizontal line in the graph, corresponding to the distance along the rail, (given by $R\theta$ in figure 3. In addition, a circular motion is superimposed, illustrated by the lines going out from the horizontal line, with a time interval of 0.1s. The resulting curved line represents the path of the rider through one period of the pendulum motion.
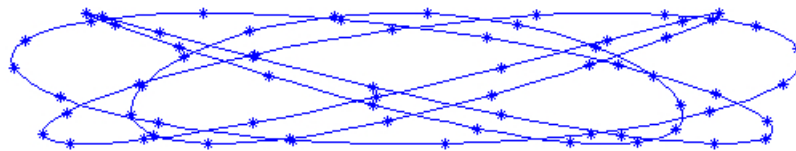
Figure 6. The figure shows the path of a rider through a few oscillations and turns. The path starts and finishes at the upper right part of the figure. An asterisk marks the position every 0.4s.
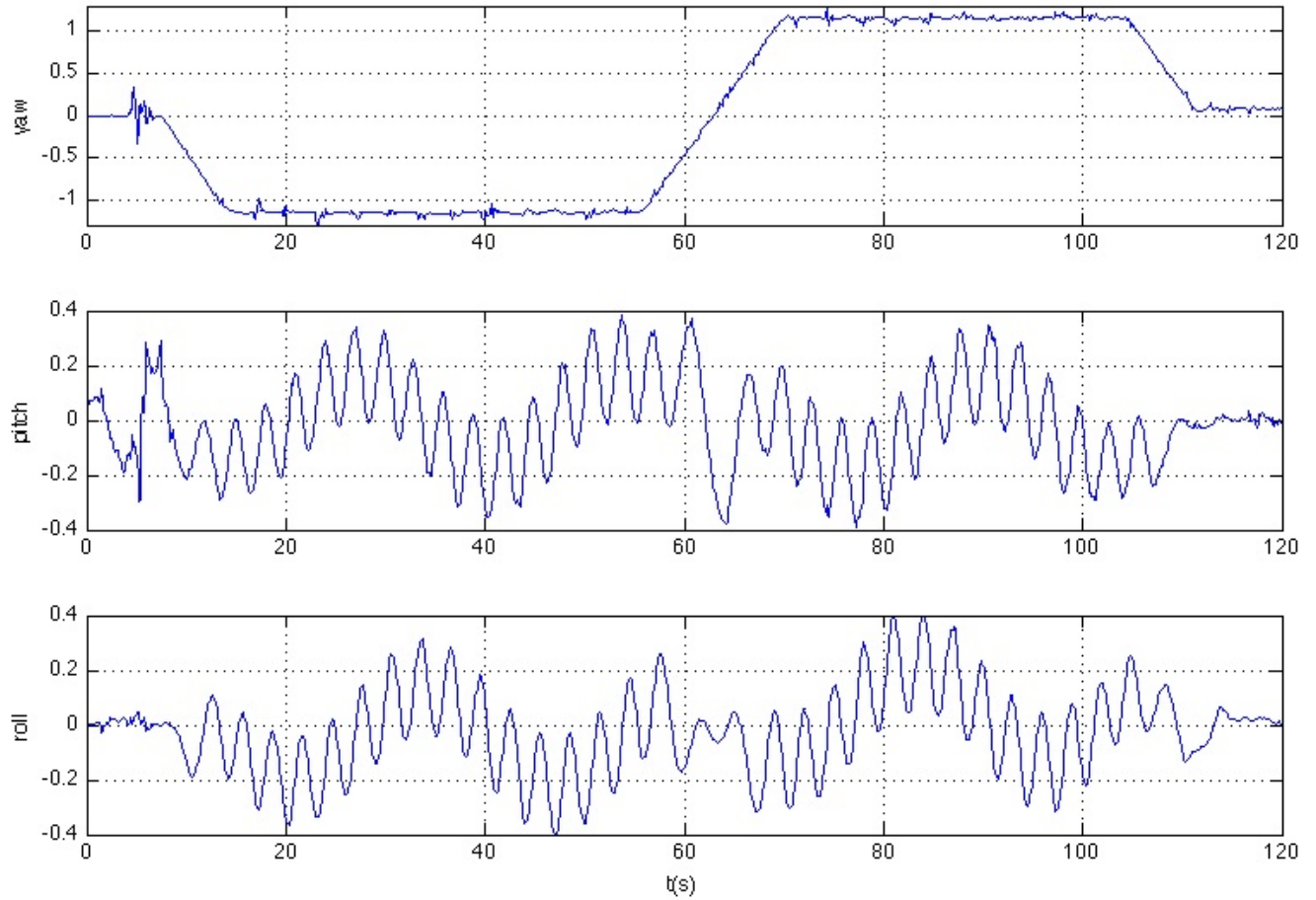
Figure 7. Measured angular velocities around the three axes of the iPhone during the ride.

4. Angular velocities during the combined motion

The analysis above uses only the accelerometer data in three dimensions. These can be obtained using many types of equipment, including the Vernier WDSS sensor [4], as described e.g. in [5] and many modern mobile phones. The possibility to record also the rotation offers new possibilities, including "motion tracking" [6], and is now available e.g. with the Iphone 4, used for the measurements presented here. Figure 6 shows the time dependence of the angular velocities around the axes of the comoving sensor. The rotation is measured around the three axes relative to the body, and are often referred to as yaw (around the "vertical" z-axis), pitch (around the "lateral" y-axis) and roll (around the "longitudinal" x axis). The yaw data shows the rotation of the boat, increasing to a constant angular velocity and then turning around to rotate in the other direction. The data are consistent with the specified rotational speed of 11 turns/minute.

The angular velocities for pitch and roll exhibit more complicated patterns. They should account for the rotations corresponding to the pendulum motion around a fixed

horizontal axis, $e_h = \sin\phi\ e_x + \cos\phi e_y$, shown in figures 2 and 4. The angle of the rotating x-y coordinate system can be written as $\phi = \Omega t$. The time dependence of the angular velocity associated with the pendulum motion can be written as $\omega_h = \omega_0 \sin pt$. However, due to the rotation of the boat (and sensor), this rotation is distributed over the x and y axes, giving

$$\omega_x = \omega_0 \sin pt \sin\phi = \omega_0 \sin pt \sin\Omega t$$
$$\omega_y = \omega_0 \sin pt \cos\phi = \omega_0 \sin pt \cos\Omega t$$

Using the properties of the trigonometric function, these expressions can be rewritten as

$$\omega_x = \frac{\omega_0}{2}\left(\cos(p-\Omega)t - \cos(p-\Omega)t\right)$$
$$\omega_y = \frac{\omega_0}{2}\left(\sin(p-\Omega)t + \sin(p-\Omega)t\right)$$

The combination of the two periodic motions results in a more rapid oscillation with angular frequency $\Omega + p$ added to a slower oscillation with an angular velocity $\Omega - p$, in this case corresponding to the periods 27.5 s and 3.0 s, respectively. These periods are clearly visible in the rotation data in figure 7.

The total horizontal angular velocity is the vector sum of these two rotations, with a modulus $|\omega_h| = \sqrt{\omega_x^2 + \omega_y^2}$ shown in bottom graph of figure 8, where the top graph shows both the rotation around the x and y axes.
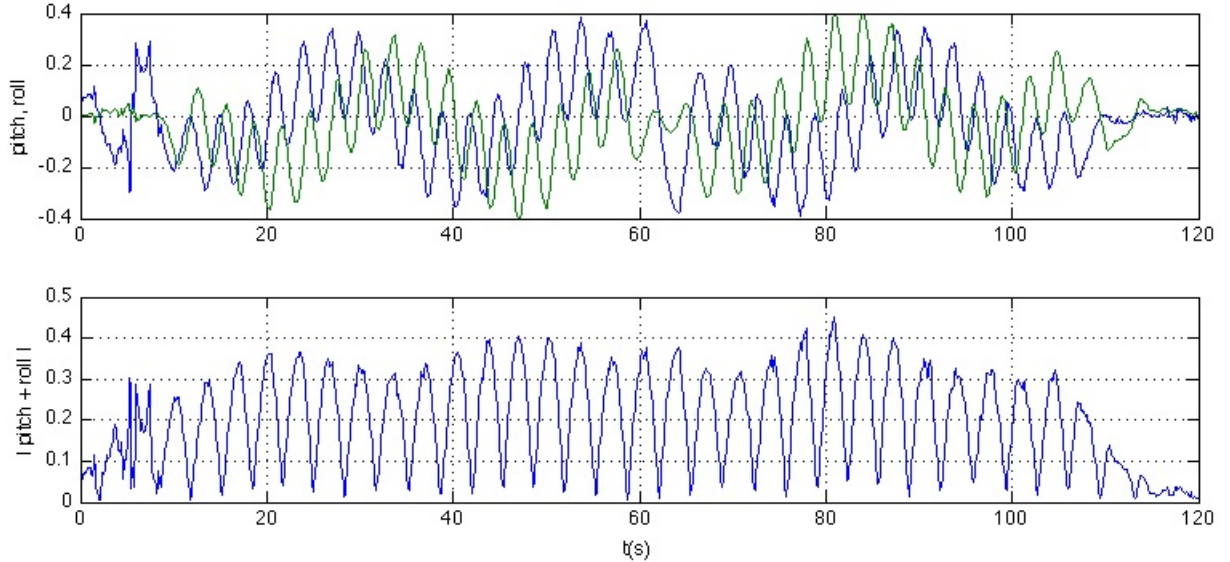
Figure 8. The "roll" and "pitch" rotations around the moving x and y axes in figure 4 are shown together in the top graph. Together they make up the rotation around the horizontal axis in the pendulum motion, and the bottom graph shows the magnitude of the vector sum in radians/s.

5. Discussion

The analysis of the acceleration and rotation data for the Rockin' Tug ride shows the richness of mathematics and physics examples exhibited already in a simple family ride. Many amusement parks feature similar rides in various sizes. An advantage of using a smaller ride is that it is easier to place the sensor so that the axes point in the desired directions. Had this not been the case, it would also have been necessary to perform an initial transformation of the coordinates. The access to rotation data, made possible e.g. by the iPhone 4, offers illuminating examples of rotations around different axes.

Detailed analyses of rides are suitable tasks for group projects in connection with amusement park visits. Amusement rides can also take the role as previous shared experiences in combination with data already available. (The dataset used in this paper is available on request, as are data for many other rides of different types.) The students may also e.g.write small programs to animate the motion of the rider, or synchronize data with video recordings of a ride. We have found that assigning 2-3 rides to groups of 4-6 students can lead to many challenging discussions of the physics involved. The learning is enhanced if the groups are required to write a report and present it orally to the other groups in the class, with different ride assignments. Asking each group to read the report of at least one other group and to prepare questions and feedback, often leads to additional reflection on their own work. The connection between the experience of forces on and in the body and the mathematical description of motion provides additional aspects and a can lead to a deepended understanding of classical mechanics.

Acknowledgements

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