

Rotating swings - a theme with variations

Ann-Marie Pendrill

National Resource Centre for Physics Education, Lund University, Box 118, SE
221 00 Lund, Sweden

E-mail: Ann-Marie.Pendrill@fysik.lu.se



Figure 1. Three StarFlyer rides: Himmelskibet at Tivoli Gardens, Copenhagen, Eclipse at Gröna Lund, Stockholm and the Praterturn in Vienna. Although the rides are very similar, the actual dimensions and forces differ.

Abstract. Rotating swing rides can be found in many amusement parks, in many different versions. The "wave swinger" ride, which introduces a wave motion by tilting the roof, is among the classical amusement rides that are found in many different parks, in different sizes, from a number of different makes and names, and varying thematization. The "StarFlyer" is a more recent version, adding the thrill of lifting the riders 60 m or more over the ground. These rotating swing rides involve beautiful physics, often surprising, but easily observed, when brought to attention. The rides can be used for student worksheet tasks and assignments of different degrees of difficulty, across many math and physics topics. This paper presents a number of variations of student tasks relating to the theme of rotating swing rides.

The Himmelskibet "Star Flyer" ride [1] in Tivoli Gardens attracts the eyes of passengers exiting Copenhagen central station, as the rotating swings on chains are raised 60 m or more into the air, and similar rides are found e.g. in Stockholm and Vienna (figure 1). This is a recent development of rotating swing rides found in many amusement parks. The wave swinger introduces also a tilt of the top, leading to a wave



Figure 2. The 48 seat wave swinger rides from Zierer at Gröna Lund, Sweden. The swings are suspended from three circles with slightly different diameters, and slightly different chain lengths. The 16 swings in the outer circle are suspended at a distance of 2 m and the four chains holding the seat are about 5.5 m long, (including the extra chain at the top and the height of the seat)



Figure 3. Tilted roof on the Zierer wave swinger at Liseberg, Sweden

motion [2] and is among the classical amusement rides found in many parks, smaller or larger, from a number of makes, with different decorative designs to fit the local park environment (figures 2-3). Smaller versions can also be found in some playgrounds. The patent classification, A63G1/28 [3] "Roundabouts ... with centrifugally-swingable suspended seats" includes 394 items, the first of which seems to be the "Flying horse machine" from 1869 by Newhall and Cummings [4].

These rotating swing rides involve beautiful physics, often surprising, but easily observed, when brought to attention. They can be used for a number of student assignments of different degrees of difficulty, across a number of math and physics topics.

Theoretical investigations for the classroom can be based on ride data, photographs, movies or accelerometer datasets. Physical model experiments can be used to develop a deeper understanding of the physics involved. Worksheets during park visits may scaffold measurements and observations for the actual amusement park ride, as well as the follow-up calculations (see e.g. [5]-[9]). Through variations in the task formulations and in the choice of data provided, the degree of difficulty can be adapted, as well as the physics or math in focus. Which physics or math topics can you identify as relevant in connection with these rides? Which questions can you ask and what tasks can be identified, using these photos? What investigations can be performed on-site or on the ride? What assignments would you like to include in a worksheet?

If I am with a group of students in a park, I try to arrive at the wave swinger ride while it is at rest, being loaded. I would ask the students to predict which swing will hang out with the largest angle; an empty swing or one with a heavy adult. Some students focus on the force of gravity making it more difficult to move the heavy adult, leading to a smaller angle. Other students focus on the larger force required to keep the larger mass moving in a circle, leading to a larger angle for the heavy adult. However, since both these effects are proportional to the mass, the angle is independent of the mass. Some students may choose a technical approach, and observe that if the angle depended on the mass, swings from an inner circle might collide with those in an outer.

Photographs, such as those in figures 1-3, can be used for classroom assignments. Can you estimate the time required for the carousel to move a full circle if you know that the central star in the Himmelskibet StarFlyer (figure 1) is 14 m across? Sufficient information, is, in principle, available, but only by making estimates from the photo and invoking previous knowledge in both math and physics. For most students, this would require significant scaffolding. The calculation requires many steps and involves geometry, as well as an understanding of acceleration and of Newton's second law. This paper presents examples of tasks of different degrees of difficulty that may be given to students, including a way to work out the relations between dimensions, angle and period of rotation. Answers to most of the tasks will be given in the text, table or figures. The paper also provide many examples of how data can be collected, analysed, processed and presented. A few suggestions are given for investigations to be performed in more extended project work. Finally, the use of worksheets and teachers roles are discussed.

1. Geometry of the ride

We first consider motion in a horizontal plane, without any tilting of the roof and use the photos to analyse the motion. The radius or diameter of the circle is always a relevant parameter in circular motions: In this case the diameter, D_m , of the circular motion of the swings in Himmelskibet can be estimated from the photo in figure 1 through comparison with the diameter, D_r of the circle of swings at rest, corresponding to the distance across the central star (figure 4).

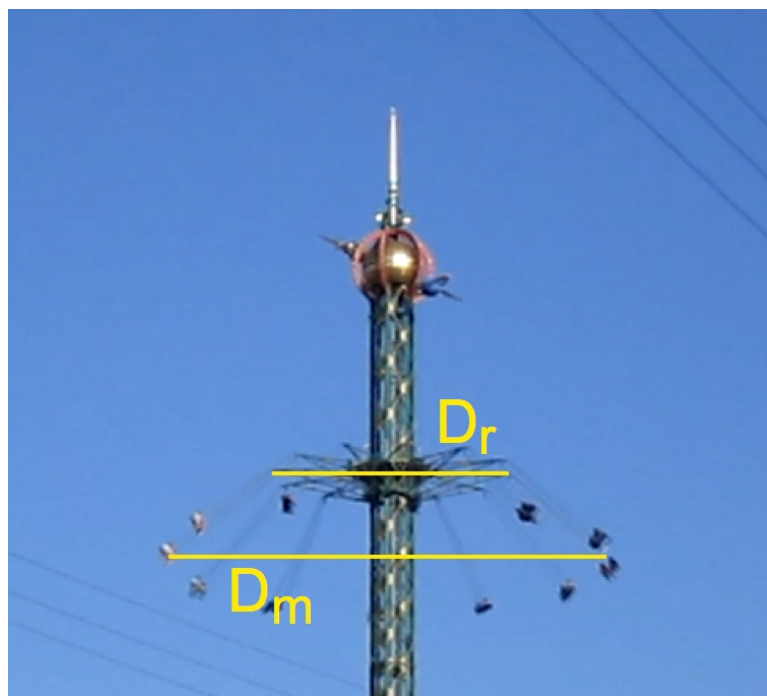


Figure 4. Using a photo of Himmelskibet to obtain the ratio $D_m/D_r \approx 1.9$ between to diameter, D_m in motion and the diameter at rest, D_r .

- (i) Measure the size of the central star and the diameter of rotation, D_m , for the swings in motion the first photo in figure 1. Use the ratio between them to estimate the diameter for the moving swings using the information that the central star is $D_r = 14$ m across.

A corresponding task for the waveswinger (figure 2) could be

- (ii) How large is the circumference of the circle of swings when the carousel is at rest? There are 16 swings, hanging at a distance of 2 m.
- (iii) How large is the diameter, D_r , of the circle of the swings when the carousel is at rest?
- (iv) How large is the diameter, D_m , when the carousel is in motion? Use the photo to compare the diameters of the suspension points in the roof and of the swings at the end of the chain?

Depending on the preparation of the students, hints may be needed on how the diameter at rest can be obtained from the circumference, which is given only implicitly.

1.1. Velocity

Velocity is another parameter of interest, which can be obtained if the time for a full circle is given (or measured). Step-by-step questions for the wave swinger could look like

- (v) How far does a swing move during a full turn of the carousel?
- (vi) What is the time, T , required for a full turn if the carousel makes 11 turns in a minute for the wave swinger?
- (vii) How fast does a swing move?

It is also possible to introduce the angular velocity, $\Omega = 2\pi/T$. For students who have taken trigonometry, a few additional questions can be asked concerning the geometry.

- (viii) Use the diameter of the motion of the swings to estimate the angle between the chains and the vertical? Use the chain length $L = 5.5\text{ m}$ for the waveswinger.
- (ix) How much does your estimate of the diameter of the circle in the wave swinger change if you take into account that you measured the circumference for a polygon with 16 sides (hexadecagon)?

The angle can be measured directly from a photo, although the perspective from the camera may lead to an exaggerated angle. Alternatively, compasses can be used to construct the situation when the end of a chain suspended at the inner radius reaches the outer radius. Again, the angle can then be measured with a protractor.

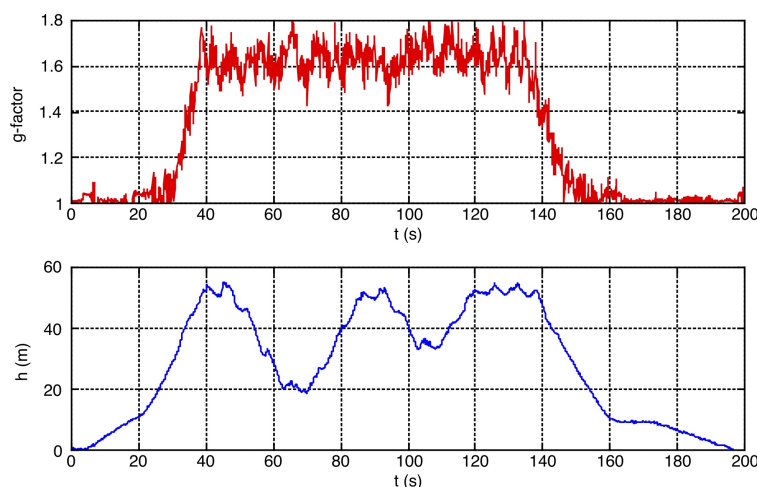


Figure 5. Accelerometer and elevation data for the Himmelskibet StarFlyer in Tivoli gardens. The elevation graph shows the ride moving up and down a couple of times during the ride, although the acceleration related to the up and down motion of the whole star is too small to show up in the g-factor. The average value of the g-factor for the main part of the ride was found to be 1.67 and is related to the angle of the chains to the normal.

2. Acceleration and force

Circular motion always involves acceleration, since the direction of motion changes all the time. A typical worksheet question may be:

- (x) The centripetal acceleration is given by the expression $a_c = v^2/r$, where $r = D_m/2$ is the radius of the circle of swings in motion. Use your estimates for speed and diameters above to estimate a_c for the wave swinger.

A teacher who has recently introduced or worked with centripetal acceleration may choose to omit the explicit expression, whereas a park worksheet typically could not build on the assumption that the expression for a_c is familiar.

None of the exercises above made a connection to the force required for an acceleration, \mathbf{a} . The body experiences a force \mathbf{X} that combines with the force of gravity, $m\mathbf{g}$, to satisfy Newton's second law $m\mathbf{g} + \mathbf{X} = m\mathbf{a}$. It is worth noting that an accelerometer, in spite of its name, does not measure acceleration, but one or more components of the vector $\mathbf{X}/m = \mathbf{a} - \mathbf{g}$. The results are often expressed in terms of a "g-factor", $G = |\mathbf{a} - \mathbf{g}|/g$ or as components of the vector $\mathbf{G} = \mathbf{X}/mg = (\mathbf{a} - \mathbf{g})/g$.

In this work we have used the Wireless Dynamic Sensor System (WDSS) [10], which, in addition to 3D accelerometer data provides elevation data, based on air pressure. Similar data can be obtained using a smartphone [11, 12, 13]. The accelerometer was mounted in a data vest, moving along with the rider. The axes of the accelerometer then rotate together with the rider, so that the "vertical" component always points in the direction of the chain, which provides the additional force, \mathbf{X} , required to provide the acceleration and to compensate for the force of gravity. Figure 5 shows the accelerometer and elevation data for the Himmelskibet Star Flyer ride.

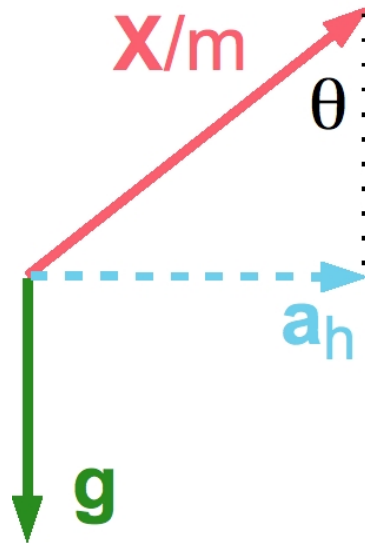


Figure 6. Freebody diagram for a horizontal acceleration, such as a chair in the wave swinger without tilted roof

The force situation in figure 6 shows that a purely horizontal acceleration is related to the acceleration of gravity as

$$a_h = g \tan \theta \quad (1)$$

for a mass suspended from a string or chain forming an angle θ to the vertical. A purely horizontal acceleration, a_h , can thus be measured by a bob on a string and a protractor (see e.g. [14] showing a protractor graded as a G-meter). The larger the acceleration, the larger the angle between the string and the vertical. For uniform rectilinear acceleration an angle of 10° corresponds to an increase of speed from 0 to 50 km/h in 8 seconds, and 20° corresponds an increase from 0 to 100 km/h in 8 seconds. An angle of 45° corresponds to an horizontal acceleration of magnitude g . However, students are likely to need prompting to realise that acceleration need not imply a change of speed.

- (xi) How can the speed remain constant, around 40 km/h, even if the acceleration may be as large as g for more than a minute?

The calculated values for the centripetal acceleration can then be inserted to estimate the angle of the chains and the forces involved: A larger acceleration requires a larger force from the string or chain: $X/m = a_h / \cos \theta$.

- (xii) Use your value above for the centripetal acceleration (and use $a_h = a_c$) to estimate the angle of the chains for the wave swinger ride.
- (xiii) How large a force does the chain exert on a chair for this angle if the combined mass of the chair and rider is m ?
- (xiv) How does the angle between the chain and the vertical depend on the mass?

These questions may be asked before the visit, but can also be used for discussions on-site, to challenge or deepen the conceptual understanding.

3. Investigations in the park

An amusement park visit gives an opportunity to focus on forces acting on the students, themselves, rather than forces they exert on other bodies.

- (xv) Close your eyes during the ride and focus on the force from the carousel acting on *you*. In which direction does that force act?

Anderson and Nashon [15] have analysed meta-cognitive strategies and discussions in a small groups of students struggling to make sense of this force situation, while visiting an amusement park.

Another investigation of the forces during the ride that may be performed on special physics days is to bring a soft mug with an inch of water into a wave swinger ride.

- (xvi) How do you expect the water surface to behave in the mug during the motion of the carousel?

As a preparation for physics days we have collected student reponses to this question, providing different alternatives. The actual result contradicts most students' expectations. Typically most respondents predict that the water level will remain horizontal - parallel to the ground. Around a third or less predict that the liquid will remain parallel to the bottom of the swing - orthogonal to the chains. The rest of

the responses are distributed over responses that the liquid leans more or less than the bottom of the swing.

During a teacher workshop in the park, some teachers performing this experiment came off the ride exclaiming that "My water didn't work" - in surprise that the water remained at the same level in the mug throughout the ride. Some teachers expressed that "nothing happened to the water - boring", and others, with more delight: "nothing happened to the water - it is amazing". An analogous experiment can also be done in an ordinary playground swing [16] or on pendulum rides (as e.g. during science days at Bakken [17]). The effect can also be demonstrated by a cordial in a bottle on a string, or in a glass suspended from a wooden triangle, as discussed in earlier work [18]. The experiment is easy but the result is surprising, challenging the understanding of what happens in accelerated motion. Students who try to explain their observation usually refer to something along the line: "The motion affects the water in the same way as it affects us", sometimes with vivid body language to illustrate how the body leans when the ride is in motion - together with the chains, swing, mug and water. A similar surprise awaits in interpreting data from an accelerometer taken along on the ride, where the vector $\mathbf{a} - \mathbf{g} = \mathbf{X}/m$, is projected onto the sensor coordinates. Since the sensor rotates together with the rider, the "vertical" component remains aligned with the spine of the rider and with the chains of the swing - and with $\mathbf{a} - \mathbf{g}$. Thus, the accelerometer data becomes non-zero only in the direction of the chain.

Table 1. Data for three rotating chair swing rides: The StarFlyers Himmelskibet in Copenhagen and Eclipse in Stockholm, and the Zierer 48 seat wave swinger Slänggungan in Göteborg, (essentially equivalent to the Kättingflygaren ride in Stockholm). The diameter and length were obtained from the data sheets, whereas the g-factor was evaluated as an average of the accelerometer data for a part of the ride moving at full speed. These results in the first group were then used to evaluate the remaining properties.

Ride		Himmelskibet	Eclipse	Slänggungan
Diameter at rest, D_r	(m)	14	20	10
Chain length, L	(m)	8	8	5.5
G - "g-factor"		1.67	1.43	1.54
$\theta = \arccos(1/G)$	(°)	53	46	50
$a_c = g \tan \theta$	(g)	1.3	1.0	1.2
$R = D_r/2 + L \sin \theta$	(m)	13	16	9
$v = \sqrt{R a_c}$	(km/h)	48	45	37
$\Omega = \sqrt{a_c/R}$	(s ⁻¹)	0.99	0.80	1.11
$T = 2\pi\Omega$	(s)	6.3	7.9	5.6
Ratio D_m/D_r		1.9	1.5	1.8

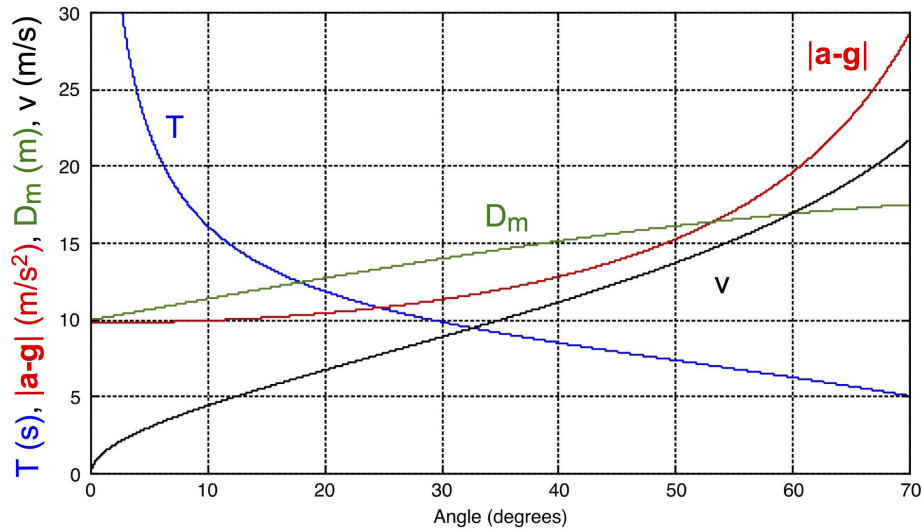


Figure 7. The relations between the diameter, D_m , speed, v , period, T , and the modulus of the vector $\mathbf{X}/m = \mathbf{a} - \mathbf{g}$ and the angle of the chains to the vertical, for the Himmelskibet StarFlyer ride. (The scale is the same for all variables, but with different units.) Note that $|\mathbf{a} - \mathbf{g}|$ depends only on the angle, but not on the ride dimensions.

4. Angle and period of rotation

As long as the swings move only in a horizontal plane, the acceleration is directly given by the angle, θ , of the chains. The angle obviously depends on the period of rotation, T , but the dependence is more easily found by calculating how the diameter, D_m in motion, the and acceleration, a_c , are related to the angle: $r = D_m/2 = D_r/2 + L \sin \theta$ and $a_c = g \tan \theta$. From the relation $a_c = r(2\pi/T)^2$ we then get an expression for the period T .

$$T = 2\pi \sqrt{\frac{r}{a_c}} = 2\pi \sqrt{\frac{D_r/2 + L \sin \theta}{g \tan \theta}} \quad (2)$$

A classroom task or a homework assignment for students in secondary school could be to draw graphs using the expressions above:

- (xvii) Make a graph showing how the acceleration, speed, period and force are related to the angle of the chains.

Drawing the graphs can be an exercise for programming e.g. in matlab or using a spreadsheet program. Figure 7 shows an example for the Himmelskibet Star Flyer ride. When a data set relating the parameters has been obtained, it can, of course, also be used to draw a graph of how the other variables depend on the period, T , of rotation.

In the park, students can measure for themselves the frequency of rotation. Moll [14] describes one of the tasks in a physics competition in Playland Amusement Park in Vancouver, BC, Canada:

- (xviii) Determine the frequency of rotation of the wave swinger while you are riding at maximum speed. Use a point form to outline the steps you followed in determining the frequency.
- (xix) You have used an experimental method to determine the frequency. What are the sources of error with your method? How can these sources of uncertainty be minimized?

The time to complete one full turn can be measured relatively easily from the ground for the wave swinger, where a person wearing e.g. a brightly coloured jacket can be easily recognized. A rotation period of $T = 5.5\text{ s}$ was obtained in this way. To obtain data relevant for different parts of the ride, I decided to bring the accelerometer on the ride again and move it sharply as the swing passes a well-defined point on the ground. The intervals Δt between these marks can be converted to a time-dependent angular frequency, $\Omega = 2\pi/\Delta t$, (possibly after some adjustments for uneven time stamps). The values for Ω obtained in this way have been inserted in the center of each time interval in the accelerometer graph for the wave swinger in figure 8, for data collected during a separate ride.

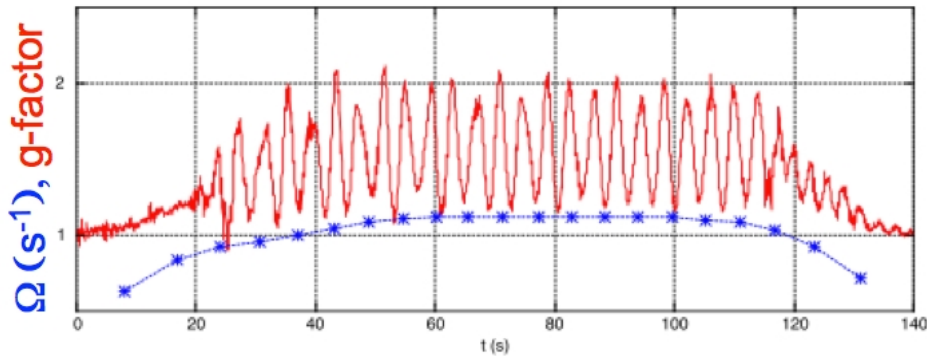


Figure 8. Accelerometer data for one of the outer swings in the wave swinger at Liseberg, together with approximate values for the angular frequency, Ω , obtained by measuring the time for each full turn.

The accelerometer data collected during the rides is an implicit measure of the angles. Table 1 shows the dimensions of the ride together with the measured values of the g-factor, evaluated as an average of the data shown in figures 5 and 8 (and similar data for the Eclipse ride in figure 1) omitting the initial and final parts. From these values, other parameters have been evaluated, as indicated above.

5. Beyond accelerometer graphs

This paper has given examples of how data collected during an amusement park visit can be used in many ways that go beyond transferring accelerometer data to a laptop and viewing the graphs. For the initial investigations discussed above a number of

approximations were introduced: "Science is the art of systematic oversimplification". Students looking for assignments for more extended projects may go into more detail. The wave swinger ride is a rich source of questions for further investigations, e.g.

- (xx) The tilt of the wave swinger roof (figure 3) introduces a wave motion, accompanied by an acceleration up and down. How does this affect the force situation?
- (xxi) What is the natural period of a 5.5 m long swing?
- (xxii) Do you expect the period of oscillation induced by the tilt to be the same as the natural period of the swing, or shorter or longer?
- (xxiii) How is the period of the swing modified by the centripetal acceleration?

6. Discussion

Teacher's preparation and follow-up activities are known to play an important role for student learning in connection with field trips [19, 20]. Although some of the observations and conceptual questions discussed in this paper involve physics that is often treated in upper secondary school, or even later, the observations themselves are still accessible for young learners, if scaffolded by discussions with a teacher or a guide in the amusement park.

Chain swing rides offer a large number of possible student assignments, from simple geometrical considerations, to surprising observations and conceptual questions, modelling and data collection and analysis. Worksheet questions can direct students' attention to details that would otherwise go unnoticed, and contribute to the connection between different ways of treating a phenomenon. The observations of the behaviour of water in a mug on a chain swing ride, or how mass does not influence the angle of the chains are both examples of the *equivalence* between inertial and gravitational mass and students could be invited to consider other situations where mass does not influence motion (such as free fall and the period of a pendulum) [21, 22]. The assignments can also introduce the idea of approximations, with possible successive refinements.

In a review of research on the use of worksheets, Kisiel [23] concludes that "simple fill-in-the-blank task completion worksheets are not effective, when every student is responsible for his or her own data, where the focus is solely to fill in the data and not to explore or participate in activities." On the other hand, worksheets were found to be quite effective when given to small groups, encouraging the students to observe, interact and discuss the concepts, and develop more connections between the concepts and the experience [23, 24, 25].

Amusement park science days can build in many different kinds of teacher roles and involvement [26]. Teachers might be invited to "Relax in our teacher hospitality room while your students immerse themselves in Education Days" [8]. However, that approach reduces the opportunities for discussions with students and involvement with their sense making of the experiences, as well as the possibility to ask challenging questions and on occasion suggest going once more on a ride or making additional measurements and

observations. In organizing our physics days in amusement parks, we aim to ensure that teachers are involved and engaged in the discussions [26]. Teachers signing up a class are expected to spend an hour close to the entrance or exit from a ride, where they can suggest, support and discuss different investigations.

References

- [1] FunTime, StarFlyer, www.funtime.com.au and Lackner H 2009, private communication
- [2] Zierer Karussell- und Spezial-maschinenbau, *Wellenflieger Produktinformation*, www.zierer.com/mainmenu/produkte/wellenflieger/
- [3] Esp@cenet, *Roundabouts ... with centrifugally-swingable suspended seats*, Patent classification A63G1/28, worldwide.espacenet.com
- [4] Newhall G L and Cummings J F 1869, *Flying-Horse Machine*, patent no US94234
- [5] Unterman N 2001 *Amusement park physics - a teacher's guide* Walch Publishing
- [6] PhysicsDay.org, *Physics, Science and Math Days at California's Great America*
- [7] NASA 2003, *Amusement park physics with a NASA twist*
{exploration.grc.nasa.gov/outreach/appd/appd_resources.html}
- [8] King's Island, www.visitkingsisland.com/student-youth-programs/youth-programs
- [9] Swedish National resource centre for physics education, *Amusement park physics*, tivoli.fysik.org
- [10] Vernier, *Wireless Dynamic Sensor System*, www.vernier.com/products/sensors/wdss/
- [11] Pendrill A-M and Rohlén J 2011 *Acceleration and rotation in a pendulum ride, measured using an iPhone 4* Phys. Educ. 46 676
- [12] Thomas B 2015 *SensorLog* <https://itunes.apple.com/se/app/sensorlog/id388014573>
- [13] Vieyra Software 2015 *Physics Toolbox Suite* <https://itunes.apple.com/se/app/sensorlog/id388014573>
- [14] Moll R 2010 *An amusement park physics competition* Phys. Educ. 45 362
- [15] Anderson D and Nashon S 2007 *Predators of knowledge construction: Interpreting students' metacognition in an amusement park physics program* Science Education 91, 298-320
- [16] Pendrill A-M and Williams G 2005 *Swings and Slides* Phys. Educ. 40 527
- [17] Bakken (*bakken.dk*) and Ulrik Lundby Hansen, private communication
- [18] Fägerlind C-O and Pendrill A-M 2015 *Liquid in accelerated motion* Phys. Educ. in print
- [19] Behrendt M and Franklin T 2014 *A Review of Research on School Field Trips and Their Value in Education* Int. J. Env. Sci. Ed., 9 235-245 available at www.ijese.com/ijese.2014.213a.pdf
- [20] deWitt J and Storksdieck M 2008 *A Short Review of School Field Trips: Key Findings from the Past and Implications for the Future* Visitor Studies 11 181
- [21] Pendrill A-M, Ekström P, Hansson L, Mars P, Ouattara L and Ryan U 2014 *The equivalence principle comes to school - falling objects and other middle school investigations* Phys. Educ. 49 425
- [22] Bagge S and Pendrill A-M 2002 *Classical physics experiments in the amusement park* Phys. Ed. 37 507
- [23] Kisiel, J 2003 *Teachers, museums, and worksheets: A closer look at learning experience* Journal of Science Teacher Education, 14, 321
- [24] Rennie L J and McClafferty T P 1995 *Using visits to interactive science and technology centers, museums, aquaria, and zoos to promote learning in science* Journal of Science Teacher Education 6 175
- [25] Mortensen M F and Smart K 2007 *Free-choice worksheets increase students' exposure to curriculum during museum visits* Journal of Research in Science Teaching 44 1389
- [26] Pendrill A-M, Kozma C and Theve A 2013 *Teacher roles during amusement park visits* Proceedings ICPE-GIREP, p 591-599, www.icpe2013.org/proceedings