Chapter 2
Acceleration and Newton's second law

This is an excerpt from Chapter 2 of the book Physics for the whole body in playgrounds and amusement park,
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1 Experiencing acceleration

What is acceleration? How can we know if we are in a car that accelerates if we close our eyes? How do you know if an elevator (lift) is going up or down when it starts? What is required for acceleration? How can we visualize acceleration - and how can it be measured?

Acceleration is at the heart of the experiences at an amusement park, not only the increase of speed, e.g. during a launch as in the photo but also as we move round and round in carousels, back and forth in large swings, and through all the different elements of roller coasters and slowing down at the end of a ride.

1.1 Images of horizontal acceleration

As an airplane starts moving on the runway, your seat pushes you forward to make you move along as the plane accelerates and the speed increases. Figure 2 shows the freebody diagram for the airplane (or a person in the airplan). The forward force for the acceleration of the whole airplane comes from the thrust of the jet engines, whereas the forward force on a person in

Fig. 1 A keyring hanging from a string during takeoff and landing of an airplane (previously published [1].)

the airplane comes from the seatback pushing you forward.

If you hold a keyring dangling from a short string during takeoff, the keyring seems to move back in the airplane, as the string has to pull the keyring forward, as in figure 1. The larger the acceleration, the larger the angle. Figure 3 shows freebody diagram for the keyring during takeoff and landing.

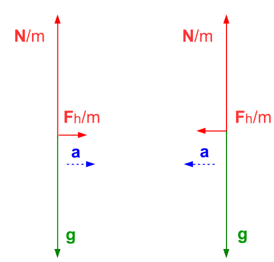
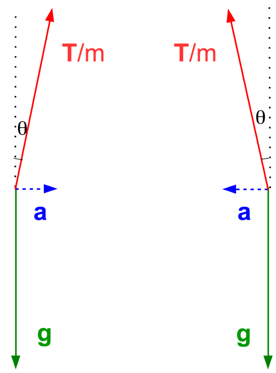


Fig. 2 Freebody diagrams for the forces acting on an airplane (or a person in an airplane) during the acceleration for takeoff and landing. The normal force (**N**) from the ground cancels the force of gravity ($m\mathbf{g}$).The horizontal force $\mathbf{F}_h$ leads to a horizontal acceleration **a**. All forces have been divided by the mass m of the accelerating body, to get the same scale, independent of mass. This convention will be used in many cases throughout the book.

A typical airplane takes off at a speed of around $v_{to} \approx 70\,\mathrm{m/s}$. If reached in in a time $t_{to} \approx 30\,\mathrm{s}$, the average acceleration is given by $a = v_{to}/t_{to} \approx 2.3\,\mathrm{m}/s^2 \approx 0.23g$. The forward force from the seat on your own body is

Fig. 3 Freebody diagrams for the forces acting on a keyring on a string during the acceleration for takeoff and landing. The forward force on the keyring, making it accelerate together with the airplane, comes from the horizontal component of the force **T** from the string. The vertical component of **T** cancels the force of gravity

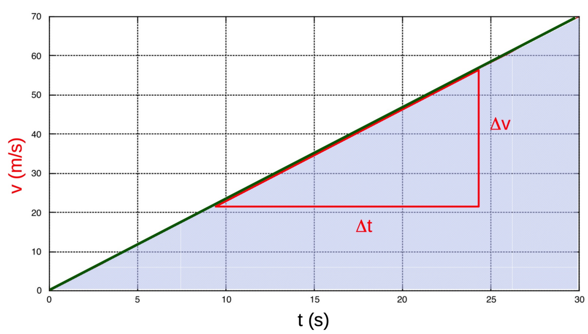


then $ma = 0.23\,mg$. Trigonometry shows that the angle for the string would be $\theta = arctan(0.23) \approx 13°$.

When the airplane slows down as it comes in for landing after reaching the ground, the seatbelt holds you back, to prevent you from continuing with your previous speed. In physics, the term acceleration denotes any change of velocity – not only speeding up, but also slowing down and/or changing direction. All changes of velocity require forces to act on the accelerating body.

An airplane usually brakes faster than it starts. This means that the magnitude of the acceleration is larger at the end of a flight, but opposing the motion. The angle of the string would also be slightly larger, but in the opposite direction.

Fig. 4 Illustrations of distance, speed and acceleration for an airplane takeoff, reaching $v = 70$ m/s in 30 s, if the acceleration is assumed to be constant.



2 Mathematical descriptions of velocity and acceleration

Textbooks often introduce motion by considering constant velocity v suddenly starting from rest at time $t = 0$. The distance travelled at a later time t can then be expressed as $s = vt$. Similarly, textbook acceleration is often uniform and starts from rest at $t = 0$, giving $v = at$ and $s = at^2/2$. These relations, represented graphically in figure 4 can be used as approximation to describe the motion during the start of an airplane.

When new university students are asked in class to recall as many relations as possible between s, v, a and t, these relations are always mentioned. Some students may come up with formulæ allowing for a non-zero velocity at $t = 0$, and starting at $s = s_0$, giving $s = s_0 + vt$, $s = s_0 + v_0 t + at^2/2$, and $v = v_0 + at$.

A discussion about what assumptions were made to obtain the kinematic formulæ, and what is required for them to hold can lead students to the more general expressions:

$$v = ds/dt \approx \Delta s/\Delta t \tag{1}$$

$$a = dv/dt \approx \Delta v/\Delta t \tag{2}$$

$$a = d^2s/dt^2 \approx \Delta(\Delta s)/(\Delta t)^2 \tag{3}$$

To find distance and velocity from acceleration, the integral expressions can be used: $s = s_0 + \int v\ dt$ and $v = v_0 + \int a\ dt$, illustrated in figure 4. For numerical data, these expressions are approximated by summation, e.g.

$$s - s_0 = \int v\ dt \approx \sum_n v_n \Delta t_n \tag{4}$$

$$v - v_0 = \int a\ dt \approx \sum_n a_n \Delta t_n \tag{5}$$

These relations are all within kinematics, where the motion is described without relation to any force.

Less known are the higher derivatives, jerk, and snap, followed by crackle and pop.

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2.1 Examples

Many examples of horizontal acceleration are found in my published papers, e.g. about launched roller coasters, chain flyer and skating wings.

Vertical acceleration examples include small and large drop towers, as well as trampoline bouncing and built-in parabolic 'flights' in roller coasters.

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3 Physics or Math? Motion is relative, acceleration is absolute.

A remarkable consequence of Newton's laws is that the physics appears the same independent of the velocity. That's why we don't notice the motion of a train or plane unless we look outside. Nor do we feel the motion of the Earth around the sun or around the centre of the galaxy.

While motion is relative, in classical mechanics acceleration is absolute. However, even in classical mechanics, an acceleration $\mathbf{a}$ can not be distinguished from a gravitational field in the opposite direction: The gravitational interaction has a very special property, as Einstein notes in is book about Relativity: The special and general theory [2] (chapter 19):

> Bodies which are moving under the sole influence of a gravitational field receive an acceleration, which does not in the least depend either on the material or on the physical state of the body.

Einstein formulated the equivalence principle: 'The gravitational mass of a body is equal to its inertial mass'. Since the gravitational force on a body is proportional to its mass m, the acceleration due to gravity is independent of mass.

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"For an observer falling freely from the roof of a house, the gravitational field does not exist." It is said that Einstein described this insight as the happiest thought of his life, when he heard about a person who had fallen from a roof and described the experience of not feeling his own weight. Einstein noted that 'our extension of the principle of relativity implies the necessity of the law of the equality of inertial and gravitational mass. Thus we have obtained a physical interpretation of this law.' On 11 July 1923, Einstein gave his Nobel lecture at Liseberg and talked about the theories of relativity [3]. The visit is commemorated with the star in the pavement outside Liseberg. What a pity that Einstein can not visit today's amusement parks and experience the weightlessness during a few seconds of free fall.

References

[1] Ann-Marie Pendrill. Understanding acceleration: An interplay between different mathematics and physics representations. *Journal of Physics: Conference Series*, 1286:012070, 2019.

[2] Albert Einstein. *Relativity: The Special and General Theory*. Henry Holt, 1920; Bartleby.com, 2000, 1920.

[3] Aant Elzinga. *Einstein's Nobel Prize, A Glimpse Behind Closed Doors: The Archival Evidence (Archive of the Nobel Museum)*. Sagamore Beach, MA, 2006.